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Now if  $\theta = \alpha$ , the plane then being parallel to one and only one element, (5) reduces to  $y^2 - 4gx\sin^2\alpha = 0$ , a parabola of latus rectum  $= 4g\sin^2\alpha$ .

If  $\theta > \alpha$  the section cuts all elements and the coefficients of  $x^2$  and  $y^2$  are both positive, and we have an ellipse whose center, axes, and eccentricity are readily found; and in particular if  $\theta = 90^\circ$ , the section is parallel to the base, the coefficients of  $x^2$  and  $y^2$  are unity and we have a circle, whose center is  $(g\sin\alpha, 0)$ . If  $\theta < \alpha$ , the section cuts both nappes, the coefficients of  $x^2$  and  $y^2$  are of unlike sign and we have a hyperbola.

If  $g = 0$ , (5) becomes  $y = \pm x \sqrt{\frac{\sin^2\alpha - \sin^2\theta}{\cos^2\alpha}} \dots (6)$ . Now if  $\theta = \alpha$ , (6) becomes  $y = 0$ , a straight line, the limit of the parabola. If  $\theta < \alpha$ , (6) represents two real straight lines, the limiting case of the hyperbola. And if  $\theta > \alpha$ , (6) represents two imaginary lines intersecting in the real point  $(0, 0)$ , which is the limiting form of the ellipse.

The equations (5) and (6) show the dependence of the nature of the conic sections upon the angle which the cutting plane makes with the axis, and the dependence of their shape upon the angle of the cone and the distance from the vertex to the first element cut.

REMARK 1. If the section be a parabola, the foot of the perpendicular from the middle point of the line through  $D$  parallel to  $BC$ , upon  $DH$ , is the focus.

REMARK 2. The eccentricity of any conic section is  $\varepsilon = [\cos\theta/\cos\alpha]$ .

## NEW AND OLD PROOFS OF THE PYTHAGOREAN THEOREM.

By BENJ. F. YANNEY, A. M., Mount Union College, Alliance, Ohio, and JAMES A. CALDERHEAD, B. Sc., Curry University, Pittsburg, Pennsylvania.

[Continued from November Number.]

LXX. Fig. 31.

$AEM = ACB$  of  $AEHC$ .

$MOL = ACB = ADK + DKBC = BHI + DKBC$ .  $LOI = BEK$ .

$\therefore ABLM \cong BCDF + AEHC$ .

LXXI. Fig. 31.

$AMPQ \cong AMOC \cong AEHC$ .  $BLPQ \cong BLOC \cong BCDF$ .

$\therefore ABLM \cong BCDF + AEHC$ .

LXXII. Fig. 31.

$MTE = BSF$ .  $\therefore BS = MT$ .  $\therefore AMLB \cong 2AMTS$ .

But  $AMTS \cong EAC + FBC$ ; since  $MTE = BSF$ , and  $AEM = ACB$ .

$\therefore 2AMTS \cong 2EAC + 2FBC \cong AEHC + BCDF$ .

$\therefore AMLB \cong BCDF + AEHC$ .

LXXIII. Fig. 31.

$$ABVU \cong ABHW. \quad UVLM \cong 2MLH \cong 2MOL + 2LOH \cong HEW + ACB + 2FBC (\cong BCDF).$$

$$\therefore ABLM \cong BCDF + AEHC.$$

LXXIV. Fig. 31.

$$ABLM \cong CZYX - 4ACB.$$

$$BCDF + AEHC = OLZH + AEHC \cong CZYX - 2MOLY \cong CZYX - 4ACB.$$

$$\therefore ABLM \cong BCDF + AEHC.$$

NOTE.—This proof is similar to that of Henry Boad's, London, 1733.

LXXV. Fig. 32.

$$ANML \cong 2ACL = 2AFB \cong AFHC.$$

$$NMKB \cong 2BCK \cong BCDE.$$

$$\therefore ABKL \cong BCDE + AFHC. \quad \text{Wipper.}$$

LXXVI. Fig. 32.

Since  $EC$  produced to  $O$  passes through the center of  $ABKL$ ,  $ABQO \cong \frac{1}{2}ABKL$ . Now,  $APCO \cong CAF$ , since  $ACO = AFP$ ;  $PBQC \cong CBE$ , since  $BCP = BEQ$ .  $\therefore ABKL \cong BCDE + AFHC$ .

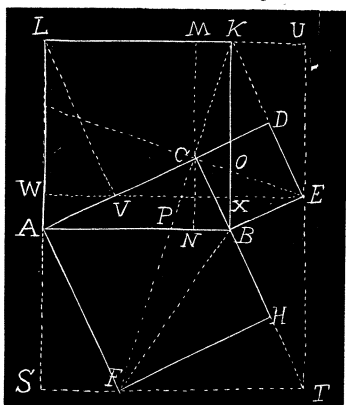


Fig. 32.

LXXVII. Fig. 32.

$$LVDK = AFHB. \quad \text{Then, } AVL = BEK.$$

$$\therefore ABKL \cong BCDE + AFHC.$$

LXXVIII. Fig. 32.

$$ABKL \cong STUL - 2ASF - 3FHT - AFHB.$$

$$BCDE + AFHC \cong STUL - 2ASF - 3FHT - LVDK (= AFHB).$$

$$\therefore ABKL \cong BCDE + AFHC.$$

LXXIX. Fig. 32.

$$ABXW \cong ABEV \cong BCDE.$$

$$WVKL \cong VEKL \cong AFHC, \quad \text{since } VED = ABC, \text{ and } VDKL = FHBA.$$

$$\therefore ABKL \cong BCDE + AFHC.$$

LXXX. Fig. 33.

$$AOML \cong AONL \cong 2ANC \cong AFHC.$$

$OBKM \cong 2BCK \cong BCDE$ , since  $SK$ , the altitude of  $BCK$ ,  $= BC$ .

$$\therefore ABKL \cong BCDE + AFHC.$$

LXXXI. Fig. 33.

$$AOML \cong 2ACL = 2AFB \cong AFHC.$$

$$OBKM \cong BKN \cong 2BCN \cong BCDE.$$

$$\therefore ABKL \cong BCDE + AFHC.$$

LXXXII. Fig. 33.

$$QTCS = PFNE.$$

$$ATL + LQK + KSB = APB + CDN + NHC.$$

$$\therefore ABKL \cong BCDE + AFHC.$$

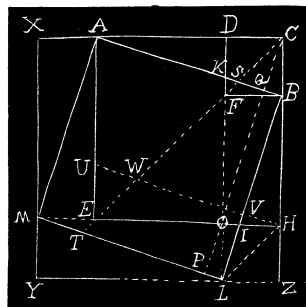


Fig. 31.

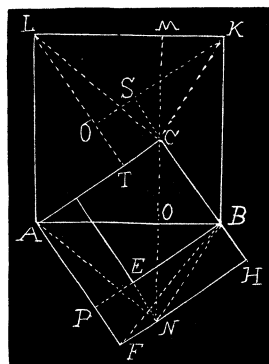


Fig. 33.